

Preface: purpose of the Assogestioni model and the taxation of Italian funds

The **purpose of the Assogestioni model** for calculating the gross share price of an Italian fund is to eliminate the effects of taxation on the net price and obtain an index (the gross price) that reflects only the results of the management of the fund. Such an index can also be used to make international comparisons between Italian funds and their untaxed counterparts in other jurisdictions.

Funds governed by Italian law are subject to a full-settlement withholding tax of 12.5% that is applied to the difference between the value of the fund's investments at the beginning and end of the year. The tax base is thus the increase in a fund's net assets, i.e. the sum of all the positive and negative changes that occur during the year (taxation of the net operating result). If a fund's net operating result is negative, it can be carried forward as a tax loss for four years. Each fund's tax liability is calculated daily and an equivalent amount set aside in a tax provision until payment is made the following year. Fund shares are consequently quoted net of taxation and holders are not required to include any gains they make in their income tax returns because the tax has already been paid on a full-settlement basis by the manager. As a general rule funds receive their investment income (i.e. capital gains, interest and dividends) gross of the full-settlement withholding tax that is normally deducted. There are three important exceptions to the taxation of funds' net operating results. The first concerns income that they receive net of 27% full-settlement withholding tax and the second tax-exempt income. In both cases the income is not subject to any further taxation and is excluded from the calculation of the net operating result subject to the 12.5% full-settlement withholding tax. The third exception concerns a particular category of income that funds receive gross and to which they have to apply a full-settlement withholding tax of 27%.

Model for the calculation of the gross share price of an Italian open-end fund

1. Introduction

Apart from Italy, most countries do not tax the results of investment funds directly but require the related income to be included in individual taxpayers' income tax returns. Setting on one side the question of the desirability of the choice made by the Italian Parliament, it is intuitive that, all other things being equal, the difference between the methods used to tax fund earnings influences the calculation of the net share price and hence the indicators of fund performance, which are based on the net value of shares.

As will be shown later, the price of an Italian fund's shares can be divided into two parts: one that depends on the activity of the manager and the other determined by the tax regime.

For the purposes of the present analysis, **we define the gross share price of an Italian open-end fund as the price at which the shares would be quoted if all the effects of the taxation of the income the fund receives gross were eliminated.** It should be noted that this definition deliberately excludes the "grossing up" of income the fund receives net of withholding taxes.

Accordingly, the determination of a fund's gross share price amounts to calculating an index of the fund's performance with all the distortionary effects of taxation¹ eliminated so that it reflects only the results achieved by the manager.

2. The model

2.1 Summary of the tax regime applicable to Italian open-end funds

In this section we describe the four different tax regimes to which the income of Italian funds may be subject. This premise is indispensable in order to identify all the ways in which the Italian tax regime can influence a fund's results.

Type A income: that received by the fund gross of tax for which the fund makes

¹ In the sense just defined.

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provision every day for the full-settlement withholding tax of 12.5% to be paid on 16 February of the following year².

The bulk of fund income is of this type, which includes: capital gains and losses on shares and bonds, share dividends, interest paid on bonds with a maturity of at least 18 months issued by companies resident in Italy with shares listed on an Italian regulated market, interest paid on Italian public-sector bonds, interest paid on bonds with a maturity of at least 18 months issued by banks, interest paid on foreign bonds with a maturity of at least 18 months, income and capital losses arising from investments in foreign UCITS (other than "historical" Luxembourg funds), interest paid on bank current accounts provided the annual average balance does not exceed 5% of the average assets under management.

Type B income: that received by the fund net of the 27% full-settlement withholding tax, including: the coupons paid on Italian bonds with a maturity of less than 18 months, interest paid on bank current accounts where the annual average balance exceeds 5% of the average assets under management and, until July 2000, the coupons paid on foreign bonds with a maturity of less than 18 months.

Type C income: that received by the fund gross of tax for which the fund makes provision every day for the full-settlement withholding tax of 27% to be paid at the end of the month in which the income became available. As of July 2000 this category consists of coupons paid on foreign bonds with a maturity of less than 18 months.

Type D income: any income that is tax exempt as far as the fund is concerned. For instance, income deriving from investments in collective investment undertakings that are subject to full-settlement withholding tax and coupons paid on bonds issued by the EIB, the EBRD, the ECSC and Euratom before 9 September 1992³.

In the light of this classification of fund income and with reference to the above definition of the gross share price, **the objective of the Assogestioni model is to calculate a fund performance index adjusted for the effects of the taxation of income of types A and C.**

² Starting in 2001 payment of the tax can be made in instalments, but this option does not alter the model's results.

³ Capital gains and losses on such securities are Type A income.

2.2 Definitions and hypotheses

Consider the cases of a generic Italian open-end accumulation fund with the following features.

The fund's shares are quoted on a daily basis and we assume that tax is the only cost they incur; in other words, we assume that there are no fees of any kind. We shall also assume that the fund is not allowed to borrow⁴.

Every day the fund calculates its tax liability with respect to type A and type C income and sets aside the appropriate amount. In the case of type A income, the fund will pay the tax liability accrued on the income earned between the first and the last day of the year in question on 16 February of the following year, or if the income is negative receive the corresponding amount. In the case of type C income, the fund must pay the tax liability accrued on the last day of the month in which the income was received (the coupons were detached).

Let N_i be the net share price and x_i the number of shares outstanding on day i ⁵. In addition, let T_i be the tax liability accrued during day i on income of types A and C. It should be noted that T_i is the amount that is set aside for tax purposes but which remains available to the manager until the day set for settlement.

For convenience we adopt the following notation, $F_i = N_{i-1}(x_i - x_{i-1})$, i.e. the flow of net fund-raising at the beginning of day i . It should be noted that F_i is the net value of the sales and redemptions made during day $i - 1$; which can be completed, however, *only* at the end of the day and *after* the fund's share price has been calculated. It is therefore as if the flow took place on day i , *before* the start of trading⁶.

The gross share price at time i will be denoted by L_i . The aim of the model described here is to determine L_i as a function of L_{i-1} , N_i , x_i and T_i .

For convenience, we define the variable ST_i as the sum of the tax liabilities accrued on

⁴ As will be shown later, these two hypotheses (i.e. no fees and no debt apart from that vis-à-vis the tax authorities) are not restrictive in the sense that the results obtained with the model developed here are independent of both the fee regime and borrowings.

⁵ N_i and x_i must be specified in the report showing the calculation of the share price that asset management companies are required to prepare daily. In passing, it should be noted that x_i is the number of shares outstanding *during* day i and therefore does not include those that will be issued/redeemed *at the end* of the day after the share price has been fixed.

⁶ Obviously, calling the flow in question F_i (because its financial effects are not felt until the morning of day i) or F_{i-1} (because it is the result of orders to buy and sell that arrived during day $i - 1$) is only a question of notation and has no effect on the results obtained.

type A and type C income but not settled at the end of day i . By definition, $ST_{i+1} = ST_i + T_{i+1} - TL_{i+1}$, where TL_{i+1} denotes the tax liability settled at the end of day $i + 1$.⁷

2.3 Calculation of the gross share price: from day 0 to day 1

As defined in the Introduction, L_i is an index of performance adjusted for the effects of the taxation of income of types A and C.

More specifically, if we consider the end of the fund's first day of life, the situation will be as follows.

At the end of day 0 (taken to coincide with the first instant of day 1) the assets available to the manager are equal to the value of the shares $x_0 N_0$ plus the taxes on income of types A and C set aside but not yet paid at time 0 ($ST_0 = 0$) plus the flow of net fund-raising between day 0 and day 1, equal to $(x_1 - x_0) N_0$. These assets have to be compared with those available at the end of day 1, which are equal to $x_1 N_1 + ST_0 + T_1$. It should be noted that L_0 can be set equal to N_0 without any loss of generality.

In other words, at the beginning of day 1 the manager receives $x_0 N_0 + (x_1 - x_0) N_0$ and at the end of the same day has $x_1 N_1 + T_1$. Given the way in which these two aggregates have been constructed, their difference measures exclusively the result achieved by the manager without the effects of the taxation of income of types A and C.

Consequently, the rate of growth, g_1 , of the gross share price measures the "management effect" and is equal to:

$$\frac{L_1}{L_0} = 1 + g_1 = \frac{x_1 N_1 + T_1}{x_0 N_0 + (x_1 - x_0) N_0} = \frac{x_1 N_1 + T_1}{x_1 N_0} \quad (1)$$

from which it follows immediately that:

$$L_1 = L_0 \frac{x_1 N_1 + T_1}{x_1 N_0} = N_0 \frac{x_1 N_1 + ST_1}{x_1 N_0}$$

It should be noted that $ST_0 = 0$ implies $T_1 = ST_1$.

2.4 From day $i - 1$ to day i

Considering the passage from the generic day $i - 1$ to the following day, i , from the

⁷ If this is the last day of the month, TL_{i+1} will be the tax liability for type C income accrued during the month that is ending. By contrast, if $i + 1$ is 16 February, TL_{i+1} will be the tax liability for type A income accrued during the previous year.

definition of the rate of growth in the gross share price

$$\frac{L_i}{L_{i-1}} = \frac{x_i N_i + T_1 + \dots + T_{i-1} + T_i}{x_i N_{i-1} + T_1 + \dots + T_{i-1}} \quad (2)$$

it follows that:

$$L_i = L_{i-1} \frac{x_i N_i + ST_{i-1} + T_i}{x_i N_{i-1} + ST_{i-1}} \quad (3)$$

2.5 At the end of the year

At 31 December of each year the share price is calculated for the last time and the definitive amount of the tax to be paid on type A income is set aside and recorded in a separate liabilities item (asset item if the income is negative).

Since the tax payables and receivables for this type of income are settled only later (in the middle of February), in the first month and a half of the following year the manager continues to possess the amount due to the tax authorities (or not to possess the amount due from them). Consequently, the formulae set out in the previous section continue to apply.

For example, on 15 January of year 2 the gross share price is given by:

$$L_{15/1/2} = L_{14/1/2} \frac{x_{15/1/2} N_{15/1/2} + ST_{14/1/2} + T_{15/1/2}}{x_{15/1/2} N_{14/1/2} + ST_{14/1/2}}$$

In this case $ST_{14/1/2}$ is the sum of: the tax liability for the previous year's type A income (not yet settled), that accrued in the current year, and that accrued for type C income but not yet settled.

2.6 Settlement of tax liabilities and the general formula

If the fund's tax liability is settled on day j (which will be 16 February for the settlement of the tax liability for the previous year's type A income and the last day of each month for the settlement of the tax liability for type C income), at the beginning of day $j + 1$ the assets available will be equal to $x_j N_j + ST_j + (x_{j+1} - x_j) N_j$ where ST_j is the *tax liability accrued but not settled on day j* , while at the end of the same day the assets available will be equal to $x_{j+1} N_{j+1} + ST_j + T_{j+1}$.

Consequently, the increase in assets achieved by the manager on day $j + 1$ compared with day j can be calculated using the formulae set out above. It is only necessary to recognize that, whereas the tax liability to be settled on day j is included in ST_{j-1} , it is no longer

present in ST_j .

The relationships set out above can easily be extended to any moment in the fund's life.

The gross share price on any day i in any year a is given by:

$$L_i = L_{i-1} \frac{x_i N_i + ST_{i-1} + T_i}{x_i N_{i-1} + ST_{i-1}} \quad (4)$$

2.7 If the fund pays fees

The model developed up to this point is based on the hypothesis that the fund pays only taxes. If this constraint is now relaxed and it is assumed that it also pays other types of charges (such as fees), the earlier results do not change insofar as, *given that the objective of the "grossing up" formula is to eliminate only the effects on the net share price of the taxation of income of types A and C*, all the other effects produced by other types of charges must be ignored. It is therefore not necessary to know whether the fund pays fees or how much they amount to. By impinging on the net share price, such charges alter the gross value but, under the hypotheses on which the model is based, this effect does not have to be eliminated. Consequently, two funds that make exactly the same investment choices but pay different fees will inevitably have two different series for their gross share price (the fund that pays higher fees will have lower gross values).

3. Comments

3.1 Analytical version of the gross share price

If it is assumed that the fund receives only income of type A (subject to the full-settlement withholding tax $\tau = 12,5\%$), it can easily be shown that the results obtained in Section 2 can be rewritten in the following form (which we call the "analytical" formula):

$$L_i = L_{i-1} \frac{\tau x_{1/1/b} (N_{1/1/b} - N_{31/12/b-1}) + \dots + \tau x_{i-1} (N_{i-1} - N_{i-2}) + x_i (N_i - \tau N_{i-1})}{\tau x_{1/1/b} (N_{1/1/b} - N_{31/12/b-1}) + \dots + \tau x_{i-1} (N_{i-1} - N_{i-2}) + x_i (N_{i-1} - \tau N_{i-1})} \quad (5)$$

where:

$$\begin{cases} b = a - 1 & i \leq 16/2 \\ b = a & i > 16/2 \end{cases}$$

In terms of **flows**, we thus have:

$$L_i = L_{i-1} \frac{x_i N_i - \tau (x_{31/12/b-1} N_{31/12/b-1} + F_{1/1/b} + \dots + F_i)}{x_i N_{i-1} - \tau (x_{31/12/b-1} N_{31/12/b-1} + F_{1/1/b} + \dots + F_i)} \quad (6)$$

and

$$T_i = \frac{\tau}{1 - \tau} x_i (N_i - N_{i-1}) \quad (7)$$

Using equation 6, it is simple to calculate the gross share price on the basis of the historical series of net share prices and the number of shares outstanding. Since these two figures are contained in the daily report showing the calculation of the share price (a public document) and the bulk of the income received by Italian funds is of type A, the "analytical" formula allows a good approximation of the gross share price to be obtained even by persons (such as individual shareholders) who do not have access to the exact values of ST_i needed to apply the "official" formula.

3.2 The net share price depends on the change in the number of shares

Returning to an issue that was mentioned in the Introduction, we use the model set out above to show exactly how taxation influences the net share price and to clarify the principle underlying the model for calculating the gross share price.

More precisely, in this section we show that taxation causes the net share price of an Italian open-end fund to depend on the number of shares outstanding. In other words, the net share price is not a time-weighted measure of the manager's performance but also depends, albeit only in part, on investors' buy and sell decisions.

At this point an analogous question arises concerning the properties of the gross share price. In the next section we shall show that, in contrast with the net share price (from which it is derived), the gross share price is independent of the number of shares outstanding and is therefore an effective time-weighted measure of the manager's performance.

Clearly, these are two good reasons for choosing the gross share price when constructing an index to measure the ability of a fund manager.

Considering the net share price and adopting the "analytical" assumption⁸, from equations

⁸ All the reasoning that follows is developed on the assumption that the fund receives only income of type A. It can be shown that when this constraint is relaxed the thrust of the results obtained is analogous (although they are slightly different and their analytical formulation is less significant).

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4 and 7 we know that:

$$\left\{ \begin{array}{l} \frac{L_i}{L_{i-1}} = 1 + g_i = \frac{x_i N_i + ST_{i-1} + T_i}{x_i N_{i-1} + ST_{i-1}} \\ T_i = \frac{\tau}{1 - \tau} x_i (N_i - N_{i-1}) \end{array} \right.$$

solving for N_i , we obtain:

$$N_i(x_i) = \underbrace{N_{i-1}(1 + g_i)}_{\text{management effect}} \underbrace{- N_{i-1}g_i\tau}_{\text{direct tax effect}} + \underbrace{\frac{g_i ST_{i-1}(1 - \tau)}{x_i}}_{\text{indirect tax effect}} \quad (8)$$

which, expressed in terms of the rate of change, $\frac{N_i}{N_{i-1}} = 1 + \pi_i$, becomes:

$$\pi_i = g_i - g_i\tau + \frac{g_i ST_{i-1}(1 - \tau)}{N_{i-1}x_i} \quad (9)$$

Equation 8 shows that the net share price on day i is the previous day's net share price revalued at the rate of growth attributable to the manager $N_{i-1}(1 + g_i)$ minus the *direct effect* of taxation on the operating result, $N_{i-1}g_i\tau$ plus $\frac{g_i ST_{i-1}(1 - \tau)}{x_i}$, which can be interpreted as the indirect effect of taxation produced by the tax liabilities accrued but still to be settled on day $i - 1$. It is the existence of this last component that causes the net share price to depend on x_i , the number of shares outstanding on day i .

This result calls for some comments:

Firstly, the net share price depends on x_i , in an inversely proportional relationship, so that, other things being equal, the higher the number of shares outstanding, the smaller the effect of a change in their number on the net share price N_i . By contrast, the effect of the flow of net fund-raising is greatest when x_i , falls rapidly towards 0 and the quantity ST_{i-1} does not decrease adequately.

Secondly, the dependence of N_i on x_i is governed by the proportionality factor $g_i ST_{i-1}(1 - \tau)$ so that in the absence of taxation ($\tau = 0$ and $ST_{i-1} = 0$) x_i does not have any influence on the net share price. In passing it should be noted that in this case equation 8 reduces to $N_{i-1}(1 + g_i)$, or, as was to be expected, the net share price varies in line with g_i and is thus a time-weighted measure of the fund's performance attributable to the manager.

Thirdly, for a given value of x_i and if $g_i > 0$, what we have called the indirect tax effect increases with the amount already set aside to meet the fund's tax liability. Intuitively, other things being equal, the larger the tax provision, the greater the assets available to the manager compared with the fund's net assets at the beginning of day i , $x_i N_{i-1}$.

For example, assuming that $x_i N_{i-1} = 10$ with $x_i = 1$ and that $ST_{i-1} = 1000$, the total assets available to the manager will be equal to 1010 and even a small increase (e.g. $g_i = +1\%$) would be reflected in an increase in the net share price of $\pi_i = +88,3\%$ (further examples are given below).

Lastly, it should be noted that when ST_{i-1}/x_i is particularly high, the fact that g_i is positive or negative, even if only slightly, has a considerable impact on the net share price (in the foregoing example if g_i had been equal to -1% , π_i would have been $-88,3\%$).

3.3 The gross share price and the flow of net fund-raising

As mentioned in the previous section, unlike the net share price, the gross share price does not depend on the number of shares outstanding; in other words, it is a time-weighted measure of the fund's performance.

In fact, under the assumption that the fund receives only income of type A, from equations 4 and 7 we have:

$$L_i(x_i) = L_{i-1} \frac{x_i N_i(x_i) + ST_{i-1} + \frac{\tau}{1-\tau} x_i (N_i(x_i) - N_{i-1})}{x_i N_{i-1} + ST_{i-1}}$$

and, taking equation 8 into account, we easily obtain:

$$L_i(x_i) = L_i = L_{i-1} (1 + g_i)$$

so that

$$\frac{dL_i}{dx_i} = 0$$

In words, the gross share price L_i is independent of changes in the number of shares outstanding, x_i . An analogous result is obtained in the more general case where the fund receives income of any type.

3.4 Some examples

In this section we give some examples of the gross and net share prices of an ideal Italian open-end fund. The aim is to illustrate the effects of the results obtained in the last two sections (the gross share price is time-weighted, while the net share price is not) and, in particular, to highlight the much greater variability of the net share price compared with the gross share price in critical situations (heavy outflows) which funds could well face in practice and which therefore deserve close attention so as to distinguish with certainty

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between the responsibility of the manager and the distortionary effects of taxation.

i	g_i	x_i	N_i	Δ_N	L_i	Δ_L
0		1	100		100	
1	10%	10	108,75	+8,75%	110	+10%
2	10%	20	118,32	+8,80%	121	+10%
3	10%	30	128,79	+8,85%	133,1	+10%
4	10%	40	140,24	+8,89%	146,41	+10%
5	65,9%	50	222,82	+58,8%	242,86	+65,9%
6	10%	80	243,12	+9,11%	267,15	+10%
7	-5%	1	189,96	-21,8%	253,79	-5%

This table shows the results of a simulation of the net and gross share prices of a fund in the first few days of its life. By definition, the gross share price varies in line with g_i , which measures exclusively the ability of the manager to generate income and, as we have seen above, is independent of the number of shares outstanding.

By contrast, the net share price follows a more up and down course owing to its partial dependence on x_i . In particular, on day 7, when there is a substantial outflow of funds (the number of shares falls from 80 to just one), while g_i is equal to -5% , the percentage change in the net share price is more than four times as big. The same thing would have occurred if g_i had been equal to $+5\%$, (in which case Δ_N would have been equal to $+21,8\%$). As was shown analytically earlier, this amplification is due to the indirect tax effect and can be measured by the ratio $ST_{i-1}/(x_i N_{i-1})$.

Since this factor distorts the ratio of the net share price to the gross share price, and in certain circumstances the distortion can be very substantial, it is worth stressing the desirability of using the gross rather than the net share price when assessing a manager's ability (and in international comparisons). Further support for this suggestion is to be found in the following example.

i	g_i	x_i	N_i	Δ_N	L_i	Δ_L	$ST_{i-1}/(x_i N_{i-1})$
0		1	100		100		
1	20%	1000	117,5	+17,5%	120	+20%	0
2	20%	2000	138,28	+17,69%	144	+20%	0,01
3	20%	3000	162,97	+17,86%	172,8	+20%	0,02
4	20%	4000	192,32	+18,01%	207,36	+20%	0,02
5	20%	100	288,62	+50,07%	248,83	+20%	1,86

When there is a massive volume of sales on day 5 (-97.5%), the manager's performance remains equal to 20% but the high value of the ratio $ST_{i-1}/(x_i N_{i-1})$, which measures the indirect tax effect, causes the net share price to jump by 50% . Judging the manager's

ability on the basis of the net share price would result in its being seriously overrated. Conversely, if g_i had been equal to 20%, Δ_N would have been equal to -50% .

3.5 Adding back fees and debt

Thanks to its flexibility, the model for calculating the gross share price presented here can be used to address the problem of "adding back" not only the taxes payable by the fund but also the fees charged by the manager.

The resulting new definition of gross share price makes it possible to calculate performance indexes adjusted for the effects of both taxation and fees. Such indexes are especially useful for institutional clients, which, as part of the process of making investment choices, can negotiate the level of fees (something that retail customers are not normally able to do).

In order to eliminate the effects of fees in the model, it is important to note that the logic underlying its construction means that it is not so much a question of distinguishing between expenses according to their nature (taxes or fees) as of specifying those that are charged to the fund on a given day but set aside for a certain period, during which they are available to the manager. In fact we have seen that since this type of expenditure is deducted immediately from the net share price but only later from the assets available to the manager it acts as a lever, in the sense that all the increases/decreases in value of this component are reflected in the net share price.

In the same way as for the taxes on income of types A and C, management fees are normally calculated daily on the total net value of the fund, set aside and settled subsequently (e.g. on the first day of each month). Accordingly, we define T'_i as the tax liability accrued during day i on income of types A and C plus the fees set aside on the same day and ST'_i as the sum of the tax liabilities accrued on type A and type C income and the fees accrued but not settled at the end of day i .

Taking these changes into account and repeating the analysis carried out to obtain equation 4, we easily arrive at the following formula for the "modified" gross share price, L'_i :

$$L'_i = L'_{i-1} \frac{x_i N_i + ST'_{i-1} + T'_i}{x_i N_{i-1} + ST'_{i-1}} \quad (10)$$

Even more generally, if we extend the definition of ST'_i to include all the fund's debts (loans granted by third parties as well as the amounts due to the tax authorities and the management company), equation 10 can eliminate the entire leverage effect due to the tax and fee levers and the "classical" lever. The tax and fee levers, as explained earlier, produce their effect as a result of the lag between the time taxes and fees are set aside and the time they are actually paid; the lag results in the fund being in debt to the tax authorities and the management company and this debt produces a leverage effect. By contrast, the "classical" leverage effect is produced when the management company deliberately borrows from third parties⁹.

Despite the theoretical interest of this adding back variant of the original model, it is worth stressing that the "official" version (equation 4) provides for the **elimination of only the effects of taxation**, since it is intended to permit meaningful comparison between funds that are subject to Italian taxation and those that are not.

3.6 When the fund distributes income

The model for calculating the gross share price can easily be extended to funds that distribute income.

The distribution of income decided by the manager takes place at the close of day j and consists in a decrease in the fund's net assets equal to the amount to be paid out, P_j per share. At the end of day j the fund's net assets amount to $x_j N_j$ and the assets to be distributed amount to $x_j P_j$. During day j the total assets available to the manager go from $x_j N_{j-1} + ST_{j-1}$ to $x_j(N_j + P_j) + ST_{j-1} + T_j$. The formula modified to take account of the distribution of income is thus:

$$L_j = L_{j-1} \frac{x_j(N_j + P_j) + ST_{j-1} + T_j}{x_j N_{j-1} + ST_{j-1}}$$

It should be noted that when $P_j = 0$, the formula returns to the normal one. It is also worth noting that the distribution of income takes place *without altering the number of shares outstanding on day j* . If some shareholders prefer to reinvest their part of the income distribution, this merely leads to an increase in the number of shares outstanding on day $j + 1$.

⁹ We are obviously assuming that funds are allowed to borrow. In practice, Italian (and European) law allows funds to borrow only for specific purposes and subject to quantitative limits.